Old and New Results on Entrepreneurship and Risk

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Abstract

In this paper we study two decisions made by an individual. First, the decision to transit from paid and secure employment into risky entrepreneurship and second, the decision about the size or scale of the venture for transitioned entrepreneurs. We show that our model conforms with the known results concerning the risk attitudes and characteristics of the decision makers to analyze the effects of greater risk aversion, wealth increases and wage increases. We then extend the literature by considering stochastic dominant shifts in the distribution of results. Interesting results arise for our comparative statics, where relative risk aversion, the DARA property, and a lower bound for risk tolerance play key roles in our results.

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1 Introduction

The theory of entrepreneurship contains an ample spectrum of possible research questions to be tackled from the economic perspective, such as, for example: the economic reasons why a worker may decide to become an entrepreneur; the way an entrepreneur may finance a new venture; the effects of entrepreneurship on economic development; or the importance of legal and economic institutions in the entrepreneurial process. For the most part, these economic analyses are based on the concepts of uncertainty, financial constraints and potential bankruptcy, and the risk characteristics associated with the preferences of entrepreneurs and workers. The reason for this is very simple: uncertainty and risk are key elements in the development of new ventures.

Knight (1921) was the first to develop the idea of the connection between entrepreneurship and risk attitudes. Knight's idea was subsequently formalized and translated into economics models by Kanbur (1979) and Kihlstrom and Laffont (1979), and since then, it has been a core component of the economic theory of entrepreneurship (Vereshchagina and Hopenhayn, 2009; Koudstaal et al., 2015). The main idea behind this theory is that the wealthy are, on average, **less risk averse** than the poor because well-behaved utility functions present the property of decreasing absolute risk aversion (DARA) and therefore, the wealthy are more prone to starting risky ventures. Furthermore, there is a second reason why (but this line of research is out of the scope of this paper) the wealthy are more likely to become entrepreneurs, and that is because they face fewer financial constraints in securing capital (Evans and Jovanovic, 1989). Consequently, we should expect to observe mostly wealthy people choosing the occupation of entrepreneurship. The poor, in contrast, are more likely to become employees for a certain, fixed wage for both of the reasons mentioned above.

Most of the recent literature that uses microeconomic models of entrepreneurship based on risk attitudes builds on the DARA assumption. Examples of the importance of this assumption are the works of Cressy (2000), van Praag and Cramer (2001), Cramer et al. (2002), Hartog et al. (2002), Kan and Tsai (2006), Ahn (2009), Caliendo et al. (2010), and Hvide and Panos (2014) among others. Our paper stands on this literature to go deeper into the problem of self-selection of occupations and entrepreneurship under risk.

In this paper we use a relatively simple model to reiterate that the traditional results (e.g. from Khilstrom and Laffont 1979) are valid, and then we extend the existing literature to look at the effect of stochastic dominant shifts in the distribution of the risk involved, and how it affects both the optimal scale of an existing business, and the decision to transition into entrepreneurship. We are able to confirm that the standard assumption in the entrepreneurship literature, positive and decreasing absolute risk aversion, is sufficient to guarantee the intuitive outcomes. However, our new results concerning stochastic dominant shifts require more than just risk aversion and DARA for the intuitive outcomes to hold.

In the next section we present the basic model of self-selection of occupation. Section 3 studies the "old" results concerning how changes in risk aversion, in outside wealth, and in the wage rate affect the optimal scale of a business, and the propensity of agents to transition into entrepreneurship rather than being employees. Section 4 is dedicated to looking at our "new" results concerning stochastic dominant changes, and, finally, section 5 concludes.

2 The Basic Model of Self-Selection of Occupations

Following the existing literature, as exemplified by Kihlstrom and Laffont (1979), we study an economy which is characterized by a single-good stochastic production function $f(L, \theta)$, where L is the labor hired and θ is a random variable indexing the state of the world and representing uncertainty in the model. We can think of θ as a random productivity shock that affects positively the production function and its marginal production, and therefore $f(L, \theta)$ and its partial derivative $f_L(L, \theta)$ are increasing in θ, and the production function satisfies $f_{LL} < 0 < f_L$. The Inada conditions are assumed to hold and therefore, an interior solution to the problem is expected. We assume that the price at which the good in question is sold is equal to 1, so that $f(L, \theta)$ also doubles as the revenue function.[1] Since the only random variable in all of the analysis is θ, all expectations are taken with respect to that variable.

Assume (for the time being) that agents have identical preferences but may differ in the initial level of wealth. They have the utility function $u(y)$, where y is the realized income. The utility function satisfies $u'' < 0 < u'$ and prudence ($u''' > 0$) coined by Kimball (1990) and widely used in models of precautionary savings and precautionary effort like Eeckhoudt et al. (2012) or Wang and Li (2014), and in models showing higher-order risk attitudes like Menegatti (2014) or Eeckhoudt et al. (2016).

Agents vary in their amount of their initial wealth a. The agents have to choose between two occupations. They can become workers and earn a certain wage w (their total final wealth in this

[1] An alternative assumption is that θ measures the price of the good.

case would be $a + w$), or they can become entrepreneurs, hiring L units of labor and earning the residual profit from a stochastic production function, which is denoted as $y(\theta) = f(L, \theta) - wL + a$, where the stochastic component is our random variable θ.

Each agent takes w as given and chooses the occupation that offers the highest utility. This result is a competitive equilibrium that translates into a partition of the set of agents into a set of workers and a set of entrepreneurs.

Let $V_E(a, w) = Eu(f(L, \theta) - wL + a))$ be the expected utility function of the entrepreneur for given wealth and wage, (a, w). And let $V_W(a, w) = u(w + a)$ be the utility function of the worker for given wealth and wage (a, w). In equilibrium, there is a wealth level $\bar{a}$ and a wage level $\overline{w}$ at which an individual is indifferent to any of the two occupations, i.e., the level of utility is the same whether the individual is a worker or an entrepreneur. We will call this decision maker the marginal or the indifferent entrepreneur. Therefore, at $(\bar{a}, \overline{w})$ we have:

$$Eu(f(L^*(\overline{w}, \bar{a}), \theta) - \overline{w}L^*(\overline{w}, \bar{a}) + \bar{a})) = u(\overline{w} + \bar{a}) \tag{1}$$

where $L^*(\overline{w}, \bar{a})$ comes from the expected utility maximization of the entrepreneur. In general, $L^*(w, a) = \underset{\{L\}}{Argmax}\, \{Eu(f(L, \theta) - wL + a))\}$, which is obtained by the following first-order condition:

$$E[u'(f(L^*(w, a), \theta) - wL^*(w, a) + a)(f_L(L^*(w, a), \theta) - w)] = 0 \tag{2}$$

Note that $\overline{w}$ in (1) corresponds to the certainty equivalent of the entrepreneur's optimal random income for the marginal agent $\bar{a}$. Then, by Jensen's inequality we know that for any risk-averse agent ($u'' < 0$), the certainty equivalent (w) is smaller than the expected value of the random variable, i.e., $w < Ef(L^*(w, a), \theta) - wL^*(w, a)$ and therefore, in our entrepreneurial context, we can say that for the marginal or indifferent entrepreneur, the expected value of his residual profits is greater than the wage received by being an employee.

Now we focus on a related but somewhat different problem, which is the problem of the optimal scale of the entrepreneurial venture, i.e. we first study the choice of the size of the projects, as measured by $L^*(w, a)$, for decision makers that have already transitioned into entrepreneurship.

3 The "old" results on optimal scale and transition

The model of Kihlstrom and Laffont (1979), which essentially resolved the choices of optimal scale of an entrepreneur's project, and the propensity of agents to become entrepreneurs rather than to be employed by entrepreneurs, is couched within an equilibrium setting in which the wage paid to employees is endogenous, and which adjusts so as to set demand and supply of labor equal. In our model, on the other hand, we assume a partial equilibrium setting in which the wage is exogenous. This allows us to consider comparative statics effects with respect to the wage. In this section, we show that the results proved by Kihlstrom and Laffont still all hold true in our model, which gives us confidence going forward when we study the effects of stochastic dominance, that our results are sufficiently robust to be of interest.[2]

Specifically, Kihlstrom and Laffont show the following (among other results):[3]

1. An increase in risk aversion will lead to a lower propensity to transition into entrepreneurship, and a smaller project for any agents engaged as entrepreneurs.

2. An increase in the risk-free wealth of an agent will, assuming decreasing absolute risk aversion, lead to a higher propensity to transition into entrepreneurship, and a larger project for any agents engaged as entrepreneurs.

3. An increase in the salary paid to employees will reduce the optimal size of entrepreneurial ventures if the utility function of entrepreneurs is DARA, or if the utility function satisfies relative risk aversion globally no larger than 1.

3.1 The scale effect

To check that these results all still hold in our model, we begin by assuming that the decision maker has committed to being an entrepreneur (i.e. he has already transitioned), and so must resolve the following entrepreneurial problem:

[2]Thus, we make no claim to originality of any of the results in this section. We include the results in an effort to confirm that they are all valid in our model.

[3]Some of these results have been found by other authors over the years, in a variety of similar theoretical models (see, for example, Cressy 2000, and Newman 2007). Others have tested the validity of these results empirically (see, for example, van Praag et al. 2002, and Kan and Tsai 2006).

$$\underset{\{L\}}{Max}\ Eu(f(L,\theta) - wL + a)$$

The first-order condition for the problem is given by equation (2) evaluated at any (a, w) pair. The second-order condition for this maximum is satisfied by the assumed concavity of both u and f in L. The resulting optimal demand for labor is a function of a and w written as $L^*(w, a)$. The scale effect is to show how optimal labor demand changes with the parameters, $L_a^*(w, a)$ and $L_w^*(w, a)$, as well as with risk aversion. For example, if $L_a^*(w, a) > 0$, then wealthier decision makers embark upon a larger scale project (as measured by employment hired in the project). We will also consider how the scale of the business is affected by an increase in risk aversion, holding (w, a) constant.

To begin with, assume that the decision maker is risk-neutral. In that case, $u'(f(L,\theta) - wL + a)$ is a constant, and the first-order condition (2) would read as

$$Ef_L(L,\theta) - w = 0$$

Notice that in this case, the optimal scale of the enterprise is independent of a. let us denote the optimal employment under risk-neutrality as $L_0^*(w)$. Clearly, this optimum simply maximizes the expected profit of the project for the risk neutral decision maker.

Now assume risk aversion, so the original first-order condition (condition (2)) applies. Since, for any two random variables $\widetilde{x}$ and $\widetilde{y}$ it holds that $E\widetilde{x}\widetilde{y} = E\widetilde{x}E\widetilde{y} + cov(\widetilde{x}, \widetilde{y})$, the first-order condition can be expressed as

$$Eu'(f(L^*,\theta) - wL^* + a)E\left(f_L(L^*,\theta) - w\right) + Cov\left(u'(f(L^*,\theta) - wL^* + a), (f_L(L^*,\theta) - w)\right) = 0$$

Consider first the term $Cov\left(u'(f(L^*,\theta) - wL^* + a), (f_L(L^*,\theta) - w)\right)$. By the initial assumptions of our model, an increase in θ will increase $f_L(L,\theta)$, so it will increase $f_L(L^*,\theta) - w$. But an increase in θ will also increase $f(L,\theta)$, and so will increase $f(L^*,\theta) - wL^* + a$, and thus, by concavity of utility $u(\cdot)$, it will decrease $u'(f(L^*,\theta) - wL^* + a)$. This implies that $Cov\left(u'(f(L^*,\theta) - wL^* + a), (f_L(L^*,\theta) - w)\right) < 0$, and consequently

$$Eu'(f(L^*,\theta) - wL^* + a)E\left(f_L(L^*,\theta) - w\right) > 0$$

Finally, since marginal utility is everywhere positive (i.e. $Eu'(f(L^*,\theta) - wL^* + a) > 0$), this implies that at $L^*(w, a)$ for a risk-averse decision maker, we must have

$$Ef_L(L^*(w,a),\theta) > w = Ef_L(L_0^*(w),\theta) \tag{3}$$

Now, since $f_{LL} < 0$, the only way that we can get inequality (3) is that $L^*(w,a) < L_0^*(w)$. So, any risk-averse decision maker will invest in a smaller scale project than a risk-neutral decision maker will (so long as both decide to become entrepreneurs rather than work as employees). This result is here captured as a lemma:

Lemma 1. *A risk averse entrepreneur will invest in smaller projects than a risk neutral entrepreneur, i.e.* $L^*(w,a) < L_0^*(w)$.

3.1.1 Scale and greater risk aversion

The above result that risk averse investors will develop a smaller scale project than risk neutral investors is suggestive that as risk aversion increases, the scale of the investment will decrease. We can study the effect of greater risk aversion by comparing the optimal scale under a given utility function with that corresponding to a more risk-averse utility function and that is what we present in our next proposition.

Proposition 1. *Increases in risk aversion induce smaller ventures for the transitioned entrepreneur.*

Proof: Recall that the final income of an entrepreneur is $y = f(L,\theta) - wL + a$. An increase in risk aversion can be studied by looking at the optimal solution under utility function $v(y) = s(u(y))$, where $s(u)$ is strictly increasing and concave, so that $s(u(y))$ is more risk averse than is $u(y)$. Denote by L_u^* the optimal labor demand when the utility function is u, that is, L_u^* satisfies (2), which we re-write here for convenience as

$$E[u'(y(L_u^*,\theta))(f_L(L_u^*,\theta) - w)] = 0$$

Now, denote by L_v^* the optimal labor demand when the utility function is $v(y) = s(u(y))$. Thus, L_v^* satisfies

$$E[s'(u(y(L_v^*,\theta))u'(y(L_v^*,\theta))(f_L(L_v^*,\theta) - w)] = 0$$

If we substitute L_u^* into the first-order condition for the problem with the more risk averse utility function, then we can evaluate the relationship between L_u^* and L_v^* by looking at the slope of the first-derivative of the more risk averse problem at L_u^*. If that slope is negative, then $L_u^* > L_v^*$, and the increase in risk aversion has reduced the optimal scale of the firm. To that end, we want to show that

$$Es'(u(y(L_u^*, \theta)))u'(y(L_u^*, \theta))(f_L(L_u^*, \theta) - w) < 0$$

We know that $s(u(y))$ is concave in y, so $s'(u(y))$ decreases with y. We also know that y increases with θ, since $\frac{\partial y}{\partial \theta} = f_\theta > 0$. Define $\widehat{\theta}$ from $f_L(L_u^*, \widehat{\theta}) - w = 0$. Since $f_{L\theta} > 0$, it happens that for all $\theta < \widehat{\theta}$ we have $f_L(L_u^*, \theta) - w < 0$, and for all $\theta > \widehat{\theta}$ we have $f_L(L_u^*, \theta) - w > 0$.

Since $s'(u(y))$ decreases with y, and y increases with θ, it is true that

$$\begin{aligned} s'(u(y(L_u^*, \theta))) &> s'(u(y(L_u^*, \widehat{\theta}))) \quad \text{for } \theta < \widehat{\theta} \\ s'(u(y(L_u^*, \theta))) &< s'(u(y(L_u^*, \widehat{\theta}))) \quad \text{for } \theta > \widehat{\theta} \end{aligned}$$

Multiply each of these inequalities by $u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w)$, which is negative with $\theta < \widehat{\theta}$ and positive with $\theta > \widehat{\theta}$,

$$\begin{aligned} s'(u(y(L_u^*, \theta)))u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) &< s'(u(y(L_u^*, \widehat{\theta})))u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) \quad \text{for } \theta < \widehat{\theta} \\ s'(u(y(L_u^*, \theta)))u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) &< s'(u(y(L_u^*, \widehat{\theta})))u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) \quad \text{for } \theta > \widehat{\theta} \end{aligned}$$

Combine these to to get

$$s'(u(y(L_u^*, \theta)))u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) \leq s'(u(y(L_u^*, \widehat{\theta})))u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) \quad \text{for all } \theta$$

Of course this would be an equality at (and only at) $\theta = \widehat{\theta}$. Now take expectations recalling that $s'(u(y(L_u^*, \widehat{\theta})))$ is a constant:

$$Es'(u(y(L_u^*, \theta)))u'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) < s'(u(y(L_u^*, \widehat{\theta})))Eu'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) = 0$$

since $Eu'(y(L_u^*, \theta))\,(f_L(L_u^*, \theta) - w) = 0$ from the first-order condition with utility function u. Therefore, an increase in risk aversion leads to a smaller optimal labor demand. **QED.**

3.1.2 Scale and effect of an increase in wealth a

Now, we are in a good position to study the effect of parameter changes on the scale of the ventures for transitioned entrepreneurs. We begin with the effect of an increase in wealth, a. Let us define

$$h(L, w, a) \equiv Ek(L, \theta) = Eu'(y(L, \theta))\,(f_L(L, \theta) - w)$$

From the first-order condition (2) we know that $h(L^*, w, a) = 0$, then applying the implicit function theorem, we know that

$$L_a^*(w,a) = -\frac{h_a(L^*, w, a)}{h_L(L^*, w, a)} \tag{4}$$

But since $h_L(L^*, w, a) < 0$ from the second order condition of the original problem, $L_a^*(w,a)$ has the same sign as $h_a(L^*, w, a)$, where

$$h_a(L^*, w, a) = Eu''\left(y(L^*, \theta)\right)\left(f_L(L^*, \theta) - w\right) \tag{5}$$

Now, we just need to work out the sign of expression (5) in order to be able to sign equation (4). Fortunately, this is not difficult:

Proposition 2. *If the entrepreneur's utility function displays DARA, an increase in the entrepreneur's wealth induces larger sizes of the ventures ($L_a^*(a,w) > 0$).*

Proof: Using the same notation as previously, we need to put a sign on

$$Eu''(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right)$$

Since the Arrow-Pratt measure of absolute risk aversion is $A(y) = -\frac{u''(y)}{u'(y)}$, we can write $h_a(L^*, w, a)$ as

$$-EA(y(L^*, \theta))u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right)$$

Use the same idea as above, that is $\widehat{\theta}$ is the value such that $f_L(L^*, \widehat{\theta}) - w = 0$. Under DARA

$$\begin{aligned} A(y(L^*, \theta)) &> A(y(L^*, \widehat{\theta})) \quad \text{for } \theta < \widehat{\theta} \\ A(y(L^*, \theta)) &< A(y(L^*, \widehat{\theta})) \quad \text{for } \theta > \widehat{\theta} \end{aligned}$$

Multiply each of these inequalities by $u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right)$, which is negative with $\theta < \widehat{\theta}$ and positive with $\theta > \widehat{\theta}$:

$$\begin{aligned} A(y(L^*, \theta))u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right) &< A(y(L^*, \widehat{\theta}))u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right) \quad \text{for } \theta < \widehat{\theta} \\ A(y(L^*, \theta))u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right) &< A(y(L^*, \widehat{\theta}))u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right) \quad \text{for } \theta > \widehat{\theta} \end{aligned}$$

That is,

$$A(y(L^*, \theta))u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right) \leq A(y(L^*, \widehat{\theta}))u'(y(L^*, \theta))\left(f_L(L^*, \theta) - w\right) \quad \text{for all } \theta$$

with equality in $\theta = \widehat{\theta}$. Now, take expectations over θ and keep in mind that $A(y(L^*, \widehat{\theta}))$ is just a number, then we get

$$EA(y(L^*,\theta))u'(y(L^*,\theta))\left(f_L(L^*,\theta)-w\right) < A(y(L^*,\widehat{\theta}))Eu'(y(L^*,\theta))\left(f_L(L^*,\theta)-w\right) = 0$$

since $Eu'(y(L^*,\theta))\left(f_L(L^*,\theta)-w\right) = 0$ from the first-order condition. So, we end up with DARA implying $-EA(y(L^*,\theta))u'(y(L^*,\theta))\left(f_L(L^*,\theta)-w\right) > 0$, which in turn indicates that $Eu''(y(L^*,\theta))\left(f_L(L^*,\theta)-w\right) > 0$, that is, L^* increases with a. **QED.**

This result succinctly proves what logic suggests, i.e. DARA is a sufficient condition for an increase in wealth for entrepreneurs induces larger sizes of the ventures ($L_a^*(a) > 0$). This result suggests that, all other things equal, entrepreneurs will be richer than employees. In a model in which the entrepreneurial decision is studied within the principal-agent framework, Newman (2007) shows that this result may not hold when the entrepreneur should also decide upon a level of effort, as well as on the optimal scale of the project.

3.1.3 Scale and effect of increase in wage w

A very clear and well-known result from elementary microeconomics of the firm under certainty is that the demand for labor is always negatively sloped, that is, if the wage rate increases, firms will demand less labor. That this result holds under risk is also often claimed in the literature on entrepreneurship. We now investigate the validity of this claim in our model, above all, if that result holds unconditionally. Interestingly, of the existing literature, the effect of a change in the wage is not commonly studied. However, Kihlstrom and Laffont (1979) do consider the slope of optimal employment in the wage as a partial step along the way to determining the equilibrium level of the wage. Here, since we assume the wage is exogenous, we are able to perform the implied comparative static directly.

Following directly what we have done for the case of an increase in initial wealth, when it is the wage that increases, the effect upon optimal firm size (as given by the labor demand) has the same sign as

$$h_w(L^*,w,a) = -L^*Eu''\left(y(L^*,\theta)\right)\left(f_L(L^*,\theta)-w\right) - Eu'\left(y(L^*,\theta)\right)$$

Since utility is everywhere increasing, the second term of this, $-Eu'\left(y(L^*,\theta)\right)$, is unambiguously negative. Therefore, it would be sufficient that the first term, $-L^*Eu''\left(y(L^*,\theta)\right)\left(f_L(L^*,\theta)-w\right)$

be non-positive. However, this term is of indeterminate sign, due to the presence of $f_L(L^*, \theta) - w$, which is positive for some θ and negative for others. That is, we require

$$-L^* Eu''\left(y(L^*, \theta)\right)\left(f_L(L^*, \theta) - w\right) \leq 0$$

Since $-L^*$ is independent of θ, and negative always (i.e. a positive amount of labor is required for the business to operate at all), then the sufficient condition for an increase in w to reduce the optimal scale of the business is

$$Eu''\left(y(L^*, \theta)\right)\left(f_L(L^*, \theta) - w\right) \geq 0 \tag{6}$$

However, notice that the expressions in (6) and in (5) are identical. Therefore, under exactly the same conditions (DARA) as for an increase in initial wealth increasing the size of the business, we can also be assured that an increase in the wage will decrease the size of the business.

Proposition 3. *When the entrepreneur's utility function displays DARA, an increase in the wage paid to employees induces smaller sizes of the ventures ($L^*_w(a, w) < 0$).*

Not-withstanding, it is still worthwhile to emphasise that we can only guarantee that an increase in the wage rate will reduce the size of the firm if we impose a condition upon the entrepreneur's utility function. That is, in general it cannot be guaranteed that higher wages lead to smaller firms.

Of course, the converse result also holds (under the same condition of DARA), that is, a decrease in the wage will induce larger business projects assuming DARA. It is also worthwhile to point out that regardless of how an increase in w affects the optimal scale of the business, such a change will always decrease the expected utility of the entrepreneur (and an decrease in the wage will be beneficial to the entrepreneur). This can be shown directly using the envelope theorem.

Expected utility in the optimal solution is

$$Eu(f(L^*, \theta) - wL^* + a)$$

The derivative with respect to w is

$$-Eu'(f(L^*, \theta) - wL^* + a)L^* < 0$$

Therefore, an increase in w leads to a less desirable situation for an entrepreneur, while a decrease in w is beneficial to an entrepreneur. This straight-forward result will be used later on.

Before moving on from the effect of an increase in the wage upon the optimal scale of the firm, it is worthwhile to provide a different sufficient condition.

Proposition 4. *If the entrepreneur's utility function displays relative risk aversion no greater than 1, then an increase in the wage rate will reduce the optimal scale of the firm ($L^*_w(a,w) < 0$).*

Proof: We would like to find a condition such that

$$-L^* Eu''\left(y(L^*,\theta)\right)\left(f_L(L^*,\theta)-w\right) - Eu'\left(y(L^*,\theta)\right) < 0$$

Start by multiplying by -1, so this is

$$Eu''\left(y(L^*,\theta)\right) L^*\left(f_L(L^*,\theta)-w\right) + Eu'\left(y(L^*,\theta)\right) > 0$$

Then move everything under a single expectations operator

$$E\left[u''\left(y(L^*,\theta)\right) L^*\left(f_L(L^*,\theta)-w\right) + u'\left(y(L^*,\theta)\right)\right] > 0$$

This is positive for sure if the function inside the expectation were to be positive for all possible θ, that is, if

$$u''\left(y(L^*,\theta)\right) L^*\left(f_L(L^*,\theta)-w\right) + u'\left(y(L^*,\theta)\right) > 0 \quad \forall\theta$$

Divide by $u'\left(y(L^*,\theta)\right) > 0$ to get

$$\frac{u''\left(y(L^*,\theta)\right)}{u'\left(y(L^*,\theta)\right)} L^*\left(f_L(L^*,\theta)-w\right) > -1 \quad \forall\theta$$

Now multiply by -1, and move the L^* term inside the bracketed term to its right:

$$A\left(y(L^*,\theta)\right)\left(f_L(L^*,\theta)L^* - wL^*\right) < 1 \quad \forall\theta$$

Then, given that $f_{LL} < 0$, if we add the (natural) assumption $f(0,\theta) = 0$ for all θ, then $f_L(L^*,\theta)L^* < f(L^*,\theta)$ for all θ. This tells us that

$$A\left(y(L^*,\theta)\right)\left(f_L(L^*,\theta)L^* - wL^*\right) < A\left(y(L^*,\theta)\right)\left(f(L^*,\theta) - wL^*\right)$$

And since $a \geq 0$,

$$\begin{aligned} A\left(y(L^*,\theta)\right)\left(f(L^*,\theta) - wL^*\right) &\leq A\left(y(L^*,\theta)\right)\left(f(L^*,\theta) - wL^* + a\right) \\ &= A\left(y(L^*,\theta)\right)\left(y(L^*,\theta)\right) \\ &= R\left(y(L^*,\theta)\right) \end{aligned}$$

where of course R is the Arrow-Pratt measure of relative risk aversion.

So, we arrive at the result that if, for all θ it holds that $R\left(y(L^*,\theta)\right) \leq 1$, then for sure

$$A\left(y(L^*,\theta)\right)\left(f_L(L^*,\theta)L^* - wL^*\right) < 1 \quad \forall\theta$$

So, it is sufficient for relative risk aversion to be less than 1 for the optimal scale of the business as measured by optimal labor demand, to be decreasing in the wage. **QED.**

Both of the above two sufficient conditions for optimal labor demand to be decreasing in the wage rate were mentioned by Kihlstrom and Laffont (1979), although they seem to have been largely ignored in the later literature.

Above all, note that from propostion 4 there is in fact no requirement for DARA so long as relative risk aversion is restricted to be no greater than 1. That is, we now have two alternative sufficient conditions, either of which alone imply the required result. The first one conditions the slope of risk aversion (DARA), while the second conditions the size of risk aversion ($R \leq 1$). Of course, some utility functions satisfy both of these conditions, but others will satisfy one or the other only, and still others will satisfy neither. DARA is often argued to be a very relevant characteristic of preferences, and it is equally argued that less risk aversion is a natural characteristic of entrepreneurs. Thus, both of the above sufficient conditions seem to make some intuitive sense.

3.2 The Transition Effect

To analyze the transition effect we will focus our attention on how the choice of occupation, between being an employee or being an employer (entrepreneur) is affected by the agent's level of risk aversion, and by small changes in the parameters of the model. To do that, we look at an agent who is, initially, indifferent between employment and being an entrepreneur, and we consider what effects lead to this agent transitioning into one or the other option. We begin with the effect of an increase in risk aversion, since the effect of an increase in wealth will imply also a change in risk aversion.

3.2.1 Transition and risk aversion

In this section we will study the effect of an increase in risk aversion for the indifferent decision maker. We can identify decision makers by a pair (u, a), where u is the utility function, and a

is initial (risk-free) wealth. A given individual i then, with (u_i, a_i), needs to decide whether to become an employee, and have final wealth of $w + a$ and utility level of $u(w + a)$, or to become an entrepreneur, and have stochastic final wealth of

$$f(L_i^*(a_i), \theta) - L_i^*(a_i)w + a_i = \widetilde{y}_i^*(a_i) + a_i$$

where $L_i^*(a_i)$ is the optimal labor choice of individual i if he becomes an entrepreneur.

Assume then that individual i is indifferent between being an employee and being an entrepreneur, then

$$Eu_i\left(\widetilde{y}_i^*(a_i) + a_i\right) = u_i(w + a_i)$$

Now consider the decision of a second individual, say individual j, who is identified by (u_j, a_j). Assume that u_j is more risk-averse than u_i, and that $a_j = a_i = a$. Due to individual j being more risk-averse, we know that there exists a function, $s(u)$, which is increasing and strictly concave, such that $u_j(z) = s(u_i(z))$. We also know that individual j will have a different optimal labor choice than will individual i, that is, if $\succ_j$ stands for the preference relation over lotteries for individual j, then for that individual we have $\widetilde{y}_j^*(a) + a \succ_j \widetilde{y}_i^*(a) + a$, and vice-versa for individual i.

By the definition above of j being more risk averse than i, and using the fact that s is strictly concave, by Jensen's inequality we get:

$$Eu_j(\widetilde{y}_j^* + a) = Es(u_i(\widetilde{y}_j^* + a)) < s\left(Eu_i(\widetilde{y}_j^* + a)\right)$$

But due to individual i having a strict preference for $\widetilde{y}_i^* + a$ over $\widetilde{y}_j^* + a$, we also know that $Eu_i(\widetilde{y}_j^* + a) < Eu_i(\widetilde{y}_i^* + a)$, which (due to the fact that s is an increasing function) implies

$$s\left(Eu_i(\widetilde{y}_j^* + a)\right) < s\left(Eu_i(\widetilde{y}_i^* + a)\right)$$

Thus, we know that

$$Eu_j(\widetilde{y}_j^* + a) < s\left(Eu_i(\widetilde{y}_i^* + a)\right)$$

Finally, since individual i is indifferent between entrepreneurship and employment, $Eu_i(\widetilde{y}_i^* + a) = u_i(w + a)$. Substituting this into the previous inequality, we can write it as

$$Eu_j(\widetilde{y}_j^* + a) < s\left(u_i(w + a)\right)$$

And, by definition, $s\left(u_i(w + a)\right) = u_j(w + a)$, so we end up with

$$Eu_j(\widetilde{y}_j^* + a) < u_j(w + a)$$

That is:

Proposition 5. *If a given individual is indifferent between employment and risky entrepreneurship, then any other individual with the same level of wealth and a more risk-averse utility function will strictly prefer employment over entrepreneurship.*

Lemma 2. *If a given individual is indifferent between employment and entrepreneurship, then any other individual with the same level of wealth and a less risk-averse utility function will strictly prefer entrepreneurship over employment.*

Proof: An individual k is less risk averse than i if $u_i(z) = s(u_k(z))$ with s strictly increasing and concave. Therefore $u_k(z) = s^{-1}(u_i(z))$ where s^{-1} is strictly increasing and convex. Using essentially the same argument as above, we have

$$Eu_k(\widetilde{y}_k^* + a) > Eu_k(\widetilde{y}_i^* + a) = Es^{-1}(u_i(\widetilde{y}_i^* + a)) > Es^{-1}(u_i(\widetilde{y}_k^* + a)) = Eu_k(\widetilde{y}_k^* + a) = u_k(w + a)$$

QED.

For the greater risk aversion effect upon transition, there is no need to invoke any condition on utility, neither DARA nor relative risk aversion in order to guarantee an unambiguous effect.

3.2.2 Increases in parameters w and a

An increase in w

Our focus here is to understand what happen with the partition set of workers and entrepreneurs when there is an exogenous change in the wage w. As we saw in our analysis of the scale effect, a change in the wage might be more complex than at first blush. However, it turns out that for the transition effect it is very intuitive.

We know that the marginal entrepreneur, given (a, w), is indifferent between working as an employee or going into entrepreneurship, that is;

$$Eu\left(f\left(L^*(w,a)\,,\theta\right) - wL^*(w,a) + a\right) = u\left(w + a\right) \tag{7}$$

It is intuitive to think that an increase in w should make an inititally marginal agent strictly prefer becoming an employee over transitioning into entreprenuership. For that to happen, we would require that an increase in w causes a larger increase in the right-hand side of (7) than in its left-hand side. However, the derivative of the right-hand side with respect to w is just

$u'(w+a)$ which is strictly positive. The derivative of the left-hand side is given by the envelope theorem from the end of the section above on optimal scale and changes in w;

$$-Eu'\left(f\left(L^*(w,a),\theta\right)-wL^*(w,a)+a\right)L^*(w,a)<0$$

Since the derivative of the right-hand side of the equation defining the marginal agent is positive and the derivative on the left-hand side is negative, we have the result that an increase in w causes an initially marginal agent to strictly prefer employment over entrepreneurship, as expected. Likewise, a decrease in the wage makes an initially indifferent person strictly prefer entrepreneurship over employment.

Intuitively, this means that an increase in the wage is a re-assigning of resources from the residual profits of the entrepreneur into the payment received by the employee, making paid employment with no risk more attractive for the (initially) indifferent decision maker.

An increase in a

The transition effect when there is a change in a looks at how the employment choice of an initially indifferent agent (as per (7)), is affected by an increase in his/her initial wealth a. The standard claim in the entrepreneurship literature is that an increase in wealth would cause an initially indifferent agent to transition into entrepreneurship, and a decrease in wealth would have the effect of causing that agent to transition into employment. In this analysis we have to assume that the wealth of the marginal agent is such that some agents transition and others do not, depending on the risk aversion characteristic of their utility function, and therefore, a partition of the set of decision makers is possible. Then, as we already know, the marginal or indifferent entrepreneur satisfies,

$$Eu\left(f\left(L^*(w,a),\theta\right)-wL^*(w,a)+a\right)=u(w+a) \tag{8}$$

In order that a marginal increase in wealth induces the decision maker to become an entrepreneur we require that the derivative of the left-hand side of (8) in a be larger than the derivative of the right-hand side of (8) in a. Using the envelope theorem for the derivative of entrepreneurial expected utility, the inequality that we require is

$$Eu'\left(f\left(L^*(w,a),\theta\right)-wL^*(w,a)+a\right)>u'(w+a) \tag{9}$$

Now, if marginal utility is convex (i.e. if the decision maker is prudent), then by Jensen's inequality we know that

$$Eu'\left(f\left(L^*(w,a),\theta\right)-wL^*(w,a)+a\right)>u'\left(Ef\left(L^*(w,a),\theta\right)-wL^*(w,a)+a\right) \tag{10}$$

and from the defining equation for the marginal investor, we know that

$$Ef\left(L^*(w,a)\,,\theta\right) - wL^*(w,a) > w$$

But then, risk aversion (decreasing marginal utility) implies that

$$u'\left(Ef\left(L^*(w,a)\,,\theta\right) - wL^*(w,a) + a\right) < u'\left(w + a\right) \tag{11}$$

so, from (10) and (11) we observe that $Eu'\left(f\left(L^*(w,a)\,,\theta\right) - wL^*(w,a) + a\right)$ and $u'\left(w+a\right)$ cannot easily be compared, even if one assumes positive prudence, so we cannot guarantee that (9) holds.

So again, refuting previous literature, the comparative static of the effect of wealth changes on the scale of projects cannot be compared using only prudence and risk aversion, and that means that we need a different approach to understand the transition effect in our entrepreneurial context, and that approach will be based on the DARA property.

Proposition 6. *When the decision maker preferences exhibit DARA, an increase in the wealth level of an initially indifferent agent guarantees that this agent transitions into entrepreneurship.*

Proof: The initially indifferent agent satisfies

$$Eu(y(L^*,\theta)) = u(w+a)$$

We know that, since utility is concave, there exists a risk-premium, $\pi(w,a)$ such that

$$u(Ey(L^*,\theta) - \pi(w,a)) = Eu(y(L^*,\theta))$$

and therefore

$$u(Ey(L^*,\theta) - \pi(w,a)) = u(w+a)$$

The requirement is then that the derivative (with respect to a) of the left-hand side of this is greater than the derivative of the right hand side:

$$u'(Ey(L^*,\theta) - \pi(w,a))\left(E\frac{\partial y(L^*,\theta)}{\partial a} - \frac{\partial \pi(w,a)}{\partial a}\right) > u'(w+a)$$

However, since $u(Ey(L^*,\theta) - \pi(w,a)) = u(w+a)$, it must also be true that $u'(Ey(L^*,\theta) - \pi(w,a)) = u'(w+a)$, and so our inequality can be written as

$$E\frac{\partial y(L^*,\theta)}{\partial a} - \frac{\partial \pi(w,a)}{\partial a} > 1$$

Since $y(L^*, \theta) = f(L^*, \theta) - wL^* + a$, we have

$$\frac{\partial y(L^*, \theta)}{\partial a} = \left(f_L(L^*, \theta) - w\right) L_a^{*\prime} + 1$$

And so, we require

$$E\left(\left(f_L(L^*, \theta) - w\right) L_a^{*\prime} + 1\right) - \frac{\partial \pi(w, a)}{\partial a} > 1$$

which is

$$E\left(f_L(L^*, \theta) - w\right) L_a^{*\prime} > \frac{\partial \pi(w, a)}{\partial a}$$

Now, it is well-known that, if the utility function displays DARA, then the risk-premium is decreasing in risk-free wealth, therefore under DARA the right-hand side of the previous equation is negative. But we also know that under risk aversion, $E\left(f_L(L^*, \theta) - w\right) > 0$, and under DARA $L_a^{*\prime} > 0$. In short, if preferences satisfy DARA, then the left-hand side of the equation above is positive and the right-hand side is negative, so the inequality is satisfied for sure. **QED.**

In conclusion, the same condition that is required for the optimal scale to increase with wealth implies that the marginal investor will transition into entrepreneurship if his wealth increases. That is, the transition effect corresponding to an increase in wealth can be guaranteed under the same conditions as the scale effect; we require DARA.

4 The "new" results: Stochastic dominance

Up to now, the underlying risk pertaining to the entrepreneur's business has been held constant, and we have considered the comparative statics with respect to risk aversion and the parameters of the problem. In that, we have followed the existing literature, and we have shown that the same results from the literature's different models all hold in our setting. However, it the study of how a change in the distribution of the risk will affect both the optimal scale of the business, and the decision to transition or not into entrepreneurship has thus far not been contemplated. To undertake that study, here we assume that the underlying density corresponding to the choice of being an entrepreneur undergoes a stochastic dominant shift. We restrict our attention to shifts of either first or second order only.

4.1 Stochastic dominance and the transition effect

Intuitively, a positive stochastic dominant shift in the density will lead to a greater propensity to switch from employment to entrepreneurship. We show now that this is indeed so.

Proposition 7. *A first or a second-order stochastic dominant shift in the distribution of results induces the indifferent decision maker to transition into risky entrepreneurship.*

Proof: Assume that initially the density is given by some probability density $g_1(\theta)$, and that given that density, an entrepreneur with wealth a establishes a firm of size $L_1^*(a)$. Then, if that individual is indifferent between employment and entrepreneurship, we would have

$$E_1 u(f(L_1^*(a), \theta) - L_1^*(a)w + a) = u(w + a)$$

where the expectation is taken with respect to $g_1(\theta)$, as is indicated by writing the expectation E_1. Now, our assumptions on utility are that it is strictly increasing and strictly concave, thus if the density changes to another one, say $g_2(\theta)$, that dominates density 1 under either first or second order stochastic dominance, then it must occur that

$$E_2 u(f(L_1^*(a), \theta) - L_1^*(a)w + a) > E_1 u(f(L_1^*(a), \theta) - L_1^*(a)w + a)$$

For this equation to be true, we only need that for first-order stochastic dominant shifts the utility function exhibits positive marginal utility, while for second-order stochastic dominant shifts we need that the utility function also exhibits risk aversion. Both assumptions are part of the initial assumptions of the model and therefore, if we allow the individual to choose the optimal scale of the firm under the new density we get;

$$E_2 u(f(L_2^*(a), \theta) - L_2^*(a)w + a) > E_2 u(f(L_1^*(a), \theta) - L_1^*(a)w + a)$$

Finally, substituting back into the original indifference equation, we now know that

$$E_2 u(f(L_2^*(a), \theta) - L_2^*(a)w + a) > E_1 u(f(L_1^*(a), \theta) - L_1^*(a)w + a)$$

that is, a stochastic dominant improvement in the density (of either first or second order), causes previously indifferent individuals to strictly prefer to become entrepreneurs. **QED.**

4.2 Stochastic dominance and the scale effect

Now assume that an individual has already transitioned into entrepreneurship, and we ask how a stochastic dominant shift in the density (again, of either first or second order), affects the optimal size of the business venture (as measured by the employment of labour). It is very tempting to assume directly that a stochastic dominant improvement in the density (of either first or second order) will lead to a larger scale of the business. Here we show that this might not be the case, or at least, to guarantee it, we need to condition the functions involved.

To study this effect, we need only consider the first-order condition for the optimal scale:

$$Eu'\left(f(L^*(a),\theta) - L^*(a)w + a\right)\left(f_L(L^*(a),\theta) - w\right) = 0$$

We can write this as the expectation of a given function equal to zero:

$$Ek(L^*(a),\theta) = 0$$

where of course, as above, $k(L^*(a),\theta) = u'\left(f(L^*(a),\theta) - L^*(a)w + a\right)\left(f_L(L^*(a),\theta) - w\right)$. We can now study the effects of stochastic dominant shifts by considering how such a change affects the value of the function $k(L^*(a),\theta)$.

4.2.1 First-order stochastic dominant shift

We already know that $k(L,\theta)$ decreases with L, from the second-order condition for an optimal choice of labor. If, additionally, we assume that $k(L,\theta)$ in increases with θ (we will see below when this assumption is valid), then a first-order stochastic dominant shift in the density must increase the expected value of $k(L^*(a),\theta)$. Therefore, a first-order stochastic dominant shift in the density will cause the expected value of $k(L^*(a),\theta)$ to become strictly positive, rather than 0 as would be required for an optimal scale. The entrepreneur will adjust L to bring the expected value of $k(L^*(a),\theta)$ back to 0, and since $k(L^*(a),\theta)$ decreases with L, this is achieved by increasing L^*. Therefore, in order for a first-order stochastic dominant shift to increase the optimal scale of the enterprise, it is sufficient that $k(L^*(a),\theta)$ is increasing in θ. This leads us to:

Proposition 8. *A first-order stochastic dominant shift in the density will increase the optimal scale of the enterprise, subject to the condition that absolute risk tolerance is sufficiently high.*

Proof: We have

$$k(L^*,\theta) = u'\left(f(L^*,\theta) - L^*w + a\right)\left(f_L(L^*,\theta) - w\right)$$

The derivative with respect to θ is

$$k_\theta = u''\left(f(L^*,\theta) - L^*w + a\right) f_\theta(L^*,\theta)\left(f_L(L^*,\theta) - w\right) + u'\left(f(L^*,\theta) - L^*w + a\right) f_{L\theta}(L^*,\theta)$$

This is the expression we require to be positive for the proposition to be true. We can go back to our more simplified notation, and write the condition as

$$u''\left(y(L^*,\theta)\right) f_\theta(L^*,\theta)\left(f_L(L^*,\theta) - w\right) + u'\left(y(L^*,\theta)\right) f_{L\theta}(L^*,\theta) > 0$$

We need to re-order this equation, but in doing so, notice that we cannot multiply or divide by $(f'_L(L^*,\theta) - w)$, since that term has ambiguous sign, and would thus affect the inequality direction in a different way depending on its sign. However, moving the utility terms to one side, and the terms involving the derivatives of the production function to the other, we find that the condition is

$$F(\theta) \equiv \frac{f_\theta(L^*,\theta)\left(f_L(L^*,\theta) - w\right)}{f_{L\theta}(L^*,\theta)} < -\frac{u'\left(y(L^*,\theta)\right)}{u''\left(y(L^*,\theta)\right)} = T(y(L^*,\theta)) \tag{12}$$

where $T(y)$ is absolute risk tolerance. **QED.**

Notice first of all that the condition required for a FOSD shift to increase the size of the business, equation (12), is in fact a relationship between functions (functions of θ), and not a condition on the values of functions. That is, the condition should be satisfied for all values of θ if the proposition is to hold true. Second, our assumption throughout everything is that $f_{L\theta} > 0$, so at small values of θ, we get $f_L(L^*,\theta) - w < 0$, and the condition will be satisfied trivially, so long as risk tolerance is positive, which is guaranteed by virtue of the assumption that the utility function is increasing and concave. The interesting cases then are when θ takes larger values, in which case it is unclear if the condition will be satisfied in general, since both sides of the inequality will be positive.

To illustrate this last proposition, assume that utility is of the (commonly assumed) hyperbolic absolute risk aversion (HARA) class, so that it has linear risk tolerance; $T(\theta) = m + by(L,\theta) = m + b\left(f(L,\theta) - wL + a\right)$, where m and b are constants. Then assume that the risk in question is in fact a risky price, so that $f(L,\theta) = \theta h(L)$, with h increasing and concave. Given those assumptions, we have $f_L = \theta h'(L)$, $f_\theta = h(L)$, $f_{L\theta} = h'(L)$, and $T'(y) = bh(L)$. Then, the condition required for a FOSD shift to increase the size of the business becomes

$$\frac{h(L)\left(\theta h'(L) - w\right)}{h'(L)} < m + b\left(\theta h(L) - wL + a\right) \tag{13}$$

The derivative of the left-hand side in θ is just $h(L)$, and the derivative of the right-hand side is $bh(L)$. Now, tolerance is everywhere positive, while the left-hand side expression is negative at small θ, so clearly the condition is satisfied for small θ. Indeed, if $b \geq 1$, tolerance grows faster than the left-hand side, so the condition is satisfied always. But if $b < 1$, then tolerance grows more slowly than the left-hand side, and at some value of θ the condition will no longer be satisfied.

It is interesting to notice that there is a relationship between this example and relative risk aversion. Concretely, under the assumptions of HARA utility and $f(L, \theta) = \theta h(L)$, the condition for a FOSD improvement to increase the size of the business, equation (13) is

$$\frac{h(L)\left(\theta h'(L) - w\right)}{h'(L)} < T(y(\theta))$$

which can be written as

$$\theta h(L) - \frac{h(L)}{h'(L)} w < T(y(\theta)). \tag{14}$$

But, since $h(L)$ is concave, $Lh'(L) < h(L)$, that is, $\dfrac{h(L)}{h'(L)} > L$. Therefore,

$$\theta h(L) - \frac{h(L)}{h'(L)} w < \theta h(L) - Lw$$

and the above condition (14) can be guaranteed if

$$\theta h(L) - Lw < T(y(\theta)) = \frac{u'(y(\theta))}{-u''(y(\theta))}$$

We can now cross-multiply so that this reads

$$\frac{-u''(y(\theta))}{u'(y(\theta))}\left(\theta h(L) - Lw\right) < 1$$

Finally, since $\theta h(L) - Lw < \theta h(L) - Lw + a = y(\theta)$, we have

$$\frac{-u''(y(\theta))}{u'(y(\theta))}\left(\theta h(L) - Lw\right) < \frac{-u''(y(\theta))}{u'(y(\theta))}\left(\theta h(L) - Lw + a\right) = R(y(\theta))$$

where of course R is the Arrow-Pratt measure of relative risk aversion. This leads us directly to:

Proposition 9. *If (i) relative risk aversion is no greater than 1, (ii) utility is within the HARA class, and (iii) $f(L, \theta) = \theta h(L)$ with h increasing and concave, then a first-order stochastic dominant shift of the density of θ will lead to a larger size of the entrepreneur's business.*

4.2.2 Second order stochastic dominant shift

The general intuition for a first-order stochastic dominant shift in the density carries over to shifts of the second order, but now the condition for the optimal scale to increase is that $k(L,\theta)$ needs to be both increasing and concave in θ as we will see below. Of course, one can make the relevant assumptions on the utility function u and the production function f such that $k(L,\theta)$ is indeed concave in θ, which will be what is required for a second-order stochastic dominant shift in the density to increase the optimal scale. Specifically, since at the optimum we have

$$k_\theta = u''\left(f(L^*,\theta) - L^*w + a\right) f_\theta(L^*,\theta)\left(f_L(L^*,\theta) - w\right) + u'\left(f(L^*,\theta) - L^*w + a\right) f_{L\theta}(L^*,\theta) > 0$$

we require $k_{\theta\theta} < 0$, that is

$$\begin{aligned} k_{\theta\theta} &= u'''(\cdot) f_\theta(L^*,\theta)^2 (f_L(L^*,\theta) - w) + u''(\cdot) f_{\theta\theta}(L^*,\theta)(f_L(L^*,\theta) - w) \\ &\quad + 2u''(\cdot) f_\theta(L^*,\theta) f_{L\theta}(L^*,\theta) + u'(\cdot) f_{L\theta\theta}(L^*,\theta) < 0 \end{aligned} \tag{15}$$

This becomes particularly arduous unless we further restrict our problem. Therefore, from here on, we only consider the case of the utility function within the HARA class (linear risk tolerance), and the production function linear in θ, that is, $f(L,\theta) = \theta h(L)$.

Specifically, we have:

Proposition 10. *Assume that u is of the HARA class, and that $f(L,\theta)$ is linear in θ, i.e. $f(L,\theta) = \theta h(L)$. Then it is sufficient, but not necessary, for a second-order stochastic dominant shift in the density of θ to increase the optimal scale of the business that the inverse of absolute prudence is sufficiently large:*

$$\frac{h(L)(\theta h'(L) - w)}{2h'(L)} < -\frac{u'''(\cdot)}{u''(\cdot)}$$

Proof: Given that the production function is linear in θ, we get $f_{\theta\theta}(L,\theta) = f_{L\theta\theta}(L,\theta) = 0$, and so (15) becomes

$$u'''(\cdot) f_\theta(L,\theta)^2 (f_L(L,\theta) - w) + 2u''(\cdot) f_\theta(L,\theta) f_{L\theta}(L,\theta)$$

This previous expression is what we require to be negative. First, divide by $f_\theta(L,\theta) > 0$, so that what is required is

$$u'''(\cdot) f_\theta(L,\theta)(f_L(L,\theta) - w) + 2u''(\cdot) f_{L\theta}(L,\theta) < 0$$

Simple operations now reveal that this is

$$\frac{f_\theta(L,\theta)(f_L(L,\theta) - w)}{2f_{L\theta}(L,\theta)} < -\frac{u''(\cdot)}{u'''(\cdot)}$$

or, since $f(L,\theta) = \theta h(L)$,

$$\frac{h(L)(\theta h'(L) - w)}{2h'(L)} < -\frac{u'''(\cdot)}{u''(\cdot)}$$

QED.

Notice that the condition required in proposition 10 is just

$$\frac{F(\theta)}{2} < -\frac{u''(\cdot)}{u'''(\cdot)}$$

where $F(\theta)$ is the function constraining risk tolerance in proposition 8 above:

$$F(\theta) = \frac{f_\theta(L,\theta)(f_L(L,\theta) - w)}{f_{L\theta}(L,\theta)}$$

To make sense of this, it would be useful to understand what the inverse of prudence is, and how it relates to risk tolerance. To that end, we have the following lemma

Lemma 3. *For any utility function, the measure of absolute prudence, $P(y)$, can be expressed as*

$$P(y) = \frac{T'(y) + 1}{T(y)}$$

where $T(y)$ is absolute risk tolerance.

Proof: Notice that

$$T'(y) = \frac{d}{dy}\left(-\frac{u'(y)}{u''(y)}\right) = -\left(\frac{u''(y)u''(y) - u'(y)u'''(y)}{u''(y)^2}\right)$$

Thus, the slope of tolerance is

$$\begin{aligned} T'(y) &= \frac{u'(y)}{u''(y)}\frac{u'''(y)}{u''(y)} - 1 \\ &= \left(-\frac{u'(y)}{u''(y)}\right)\left(-\frac{u'''(y)}{u''(y)}\right) - 1 \\ &= T(y)P(y) - 1 \end{aligned}$$

where $P(y)$ is absolute prudence. Therefore, we get

$$P(y) = \frac{T'(y) + 1}{T(y)}$$

QED.

Notice that, from Lemma 3, we know that the inverse of prudence is directly related to risk tolerance by

$$\frac{1}{P(y)} = \frac{T(y)}{T'(y)+1}$$

And since, under the HARA class of utility functions, tolerance is linear, then $T'(y) = b$ where b is a constant. So, with HARA utility, the inverse of prudence is a linear function of tolerance, and it is everywhere less than tolerance (assuming, of course DARA, or $T'(y) > 0$). Given this, we can now state the following, which follows directly from our previous results:

Lemma 4. *Assuming HARA utility, with the slope of risk tolerance equal to a constant b, and if the production function is linear in θ, then a second order stochastic dominant shift in the density of θ will increase the optimal scale of the business if*

$$F(\theta) < \frac{2}{b+1}T(y)$$

If, for example, $b = 1$, then we find that exactly the same condition guarantees that either a first or a second order stochastic dominant shift results in an increase in the size of the business. More specifically, using the same argument as in proposition 9 above, we can establish a sufficient condition concerning the effect of SOSD and relative risk aversion:

Proposition 11. *Assuming HARA utility, with the slope of risk tolerance equal to a constant b, and assuming that the production function is linear in θ, then a second order stochastic dominant shift in the density of θ will increase the optimal scale of the business if relative risk aversion is not greater than $\frac{2}{b+1}$.*

Proof: From Lemma 4 above, write the sufficient condition for a second-order stochastic dominant shift to increase the scale of the business as

$$\frac{h(L)(\theta h'(L) - w)}{h'(L)} < \frac{2}{b+1}T(y) \quad \Longrightarrow \quad h(L)\theta - \frac{h(L)w}{h'(L)} < \frac{2}{b+1}T(y)$$

Given that the production function, $h(L)$ is concave, then $\frac{h(L)}{h'(L)} > L$, so $h(L)\theta - \frac{h(L)w}{h'(L)} < h(L)\theta - Lw + a = y$. Therefore, our condition is guaranteed if $y \leq \frac{2}{b+1}T(y)$, which we express as

$$\frac{y}{T(y)} \leq \frac{2}{b+1} \quad \text{i.e.} \quad R(y) \leq \frac{2}{b+1}$$

where again $R(y)$ is relative risk aversion. **QED.**

If, for example, $b \leq 1$, then we would have $\frac{2}{b+1} \geq 1$, and in this case both of the sufficient conditions (that for FOSD and that for SOSD) for optimal scale to increase under a stochastic dominant shift in the density are satisfied whenever relative risk aversion is no greater than 1.

5 Concluding Remarks

We have revisited the literature on the choices of economic agents regarding entrepreneurship. Specifically, we have considered both the propensity for individuals to become entrepreneurs rather than to be employed by an entrepreneur, and we have considered the optimal size of the business that an entrepreneur will establish. We have shown that the model we have assumed fulfills all of the known results from the existing literature, that is, (i) greater (resp. lower) risk aversion leads to a lower (resp. greater) propensity to transition into entrepreneurship, and to lower (resp. greater) size of business for any transitioned entrepreneur; (ii) under an assumption of DARA, an increase (resp. decrease) in risk free wealth leads to a greater (resp. lower) propensity to become an entrepreneur, and to a larger (resp. smaller) entrepreneurial venture. We have also shown that an increase (resp. decrease) in the (exogenous) wage paid to employees will decrease (resp. increase) the propensity to become an entrepreneur, and assuming either DARA or relative risk aversion less than 1, it will lead to smaller (resp. larger) entrepreneurial ventures.

Second, we extend the literature to consider how stochastic dominant shifts in the underlying entrepreneurial risk affect both transition and scale. We found that transition follows the expected intuition, that is, a stochastic dominant shift of either first or second order, will increase the propensity to become an entrepreneur. However, the scale effect is much more interesting under stochastic dominance. There, we find that in order to guarantee the intuitive outcome, namely that a stochastic dominant shift increases the optimal scale of the venture, it is necessary to condition risk tolerance to be sufficiently high. To illustrate that result, we have appealed to the case of HARA utility and a risk that enters the decision making process linearly. In that case, the FOSD effect can be guaranteed if relative risk aversion is no greater than 1. The analysis of a SOSD shift is far more complex, and we have limited our results concerning that case to when utility is in the HARA class, and the risk enters the problem linearly. We find that the same threshold for the condition on FOSD appears again, but this time in relation to the inverse of absolute prudence. We also show that, in the special case of HARA and linear risk, our sufficient conditions for stochastic dominance can be expressed in terms of relative risk aversion being no greater than 1.

Our analysis suggests plenty of scope for extentions. Mainly, it would be of interest to resolve the issue of SOSD generally, that is, without having to appeal to HARA utility and a linear risk. We could easily express the condition that is required (that the first derivative of utility with respect to labor should be concave in the risk), but analysing what that actually implies for the shapes of utility and the production/revenue function is very complex.

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